4. PERMUTATIONS AND COMBINATIONS

PREVIOUS EAMCET BITS

1. The number of ways in which 13 gold coins can be distributed among three persons such that each one gets at least two gold coins is **[EAMCET-2000]**

   1) 36 2) 24 3) 12 4) 6

   Ans : 1

Sol : Required number of ways

= coefficient. x^{13} in $(x^2+x^3+....)^3$

= coefficient. x^7 in $(1+x+x^2+....)^3$

= coefficient. x^7 in $(1+x)^{-3}$

= ${}^9C_7 = {}^9C_2 = 36$

2. If C (2n, 3) : C (n, 2) = 12 : 1, then n = **[EAMCET-2000]**

   1) 4 2) 5 3) 6 4) 8

   Ans: 2

Sol : ${}^{2n}C_3 : {}^nC_2 = 12 :1$

${}^{2n}C_3 = {}^{12n}C_2.{}^n{}_{c2}$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{6} = 12.\frac{n(n-1)}{2}$$

$\Rightarrow 2n-1 = 9 \Rightarrow n = 5$

3. The number of quadratic expressions with the coefficients drawn from the set{0, 1, 2, 3} is

   1) 27 2) 36 3) 48 4) 64 **[EAMCET-2000]**

   Ans : 3

Sol : $ax^2+bx+c = 0$

a can be filled in 3 ways

b can be filled in 4 ways

c can be filled in 4 ways

Required no. of ways = 3 x 4 x 4 = 48

4. The number of ways in which 5 boys are 4 girls sit around a circular table so that no two girls sit together is **[EAMCET-2001]**

   Ans : 1

   1) 5! 4! 2) 5! 3! 3) 5! 4) 4!

Sol : First we arrange 5 boys around a circle in (5-1)! = 4! Ways then we have 5 gaps between them then arrange 4 girls in 5 gaps arrangement of 4 girls in 5 gaps

Arrangement of 4 girls in 5 gaps = ${}^5P_4$ = 5!

$\therefore$ Required no. of ways = 5! 4!

5. Using the digits 0, 2, 4, 6, 8 not more than once in any number, the number of 5 digited numbers that can be formed is **[EAMCET-2001]**

1) 16 2) 24 3) 120 4) 96

Ans: 4

Sol : Required no. of ways = 5! – 4! = 120-24 = 96

6. If n and r are integers such that $1 \le r \le n$ then n . C (n-1, r-1) = **[EAMCET-2002]**

1) C (n, r) 2) n . C (n, r) 3) r C (n, r) 4) (n - 1) . C (n, r)

Ans: 3

Sol : $n.c(n-1, r-1) = n.(n-1)c_{r-1}$

$$= n.\frac{(n-1)!}{(r-1)!(n-r)!} \times \frac{r}{r}$$

$$= \frac{n!r}{r!(n-r)!} = r.^{n}c_{r}$$ = r. c(n,r)

7. The least value of the natural number 'n' satisfying c(n,5) + c(n,6) > c(n+1,5) **[EAMCET 2002]**

1) 10 2) 12 3) 13 4) 11

Ans: 1

Sol : Given $^{n}c_5 + ^{n}c_6 > ^{(n+1)}c_5$

$^{(n+1)}c_6 > ^{(n+1)}c_5$

$$\frac{(n+1)!}{6!(n-5)!} > \frac{(n+1)!}{5!(n-4)!}$$

$\Rightarrow n > 10$

$\therefore$ The least value of 'n' is 10

8. The no. of ways such that 8 beads of different colour be strung in a neckles is...**[EAMCET-2002]**

1) 2520 2) 2880 3) 4320 4) 5040

Ans: 1

Sol : Required number of ways = $\frac{(8-1)!}{2}$ = 2520

9. The number of 5 digited numbers which are not divisible by 5 and which contains of 5 odd digits is **[EAMCET-2002]**

1) 96 2) 120 3) 24 4) 32

Ans: 1

Sol : The 5 odd digits be 1,3,5,7,9

Required = 5! – 4!

= 120 – 124

= 96

10. Let l_1 and l_2 be two lines intersecting at P. if A_1, B_1, C_1 are points on l_1, and A_2, B_2, C_2, D_2, E_2 are points on l_2 and if none of those coincides with P, then the number of triangles formed by these eight points. **[EAMCET-2003]**

1) 56 2) 55 3) 46 4) 45

Ans: 4

Sol : If triangle is including point P the other points must be one from l_1 and other point from l_2, Number of triangles formed with P.

$n(E_1) = {}^3c_1 \times {}^5c_1 = 15$

When p is not included

Number of triangles formed

$n(E_2) = {}^3c_2 \times {}^5c_1 + {}^3c_1 \times {}^5c_2$

= 15 + 15

= 30

$\therefore$ Total number of triangles = $n(E_1) + n(E_2)$

= 15 + 30

= 45

11. The number of positive odd divisors of 216 is **[EAMCET-2004]**

1) 4 2) 6 3) 8 4) 12

Ans: 1

Sol: The factors of 216 = $2^3 . 3^3$

The odd divisors are the multiplied 3.

$\therefore$ The number of positive odd divisors

= 3 + 1 = 4

12. A three digit number n is such that the last two digits of it are equal and different from the first. The number of such n's is **[EAMCET 2005]**

1) 64 2) 72 3)81 4)900

Ans: 3

Sol: If the last two digits are equal to then the first digit may 1 to 9

If the last two digits are equal to 1 to 9 then the first digit may be selected in 8 ways.

$\therefore$ The required number = 9 + 9 $\times$8

= 81

13. If N denotes Set of all positive integers and if and if is defined by the sum of positive divisors of . then where is a positive integer is **[EAMCET-2005]**

1) $2^{k+1}-1$ 2) $2\left(2^{k+1}-1\right)$ 3) $3\left(2^{k+1}-1\right)$ 4) $4\left(2^{k+1}-1\right)$

ns: 3

Sol: Given f(x) = the sum of positive divisors of n

$$f\left(2^k.3\right) = 3\left(1+2+2^2+3^3+...+2^k\right)$$

$$= 3\left(\frac{1\left(2^{k+1}-1\right)}{2-1}\right)$$

$$= 3\left(2^{k+1}-1\right)$$

14. The number of natural numbers less than 1000, in which no two digits are repeated is

[EAMCET 2006]

1) 738 2) 792 3) 837 4) 720

Ans: 1

Sol : The number of 1 digit numbers = 9

The number of 2 digit numbers = 9 x 9 = 81

The number of 3 digit numbers = $9 \times 9 p_2 = 648$

$\therefore$ The number of Required numbers

= 9 + 81+ 648 = 738

15. The number of ways of arranging 8 men and 4 women around a circular table such that no two women can sit together, is **[EAMCET-2007]**

Ans:

1) 8! 2) 4! 3) 8! 4! 4) $7!.{}^8P_4$

Ans: 4

Sol: Number of ways of arranging 8 men around a circle = (8-1)! = 7!

Then we have 8 gaps between them

Number of ways of arranging 4 women in 8 gaps = ${}^8p_4$

$\therefore$ Required number of ways = 7!. ${}^8p_4$

16. If a polygon of n sides has 275 diagonals, then n = **[EAMCET-2007]**

1) 25 2) 35 3) 20 4) 15

Ans: 1

Sol: Number of diagonals of a polygon of n sides = 275

$$\frac{n(n-3)}{2} = 275$$

$$n(n-3) = 550$$

$$n(n-3) = 25 \times 22$$

$\therefore$ n = 25

17. 9 balls are to be placed in 9 boxes, and 5 of the balls can not fill into 3 small boxes. The numbers of ways of arranging one ball in each of the boxes is **[EAMCET-2008]**

1) 18720 2) 18270 3) 17280 4) 12780

Ans: 3

Sol : 5 balls can be placed in 6 boxes (other than the 3 small boxes) in ${}^6p_5$ ways

The remaining 4 balls can be placed in the remaining 4 boxes in 4! ways.

$\therefore$ The required number of arrangements = ${}^6p_5 \times 4!$

18. If ${}^np_r = 30240$ and ${}^nc_r = 252$ then the ordered pair (n,r) =

1) (12,6) 2) (10,5) 3) (9,4) 4) (16,7)

Ans: 2

Sol : $\frac{{}^np_r}{{}^nc_r} = \frac{30240}{252}$

$\Rightarrow r! = 120$

$\Rightarrow r! = 5!$

$\Rightarrow r = 5$

${}^np_5 = 30240 = {}^{10}p_5 \Rightarrow$ n = 10

$\therefore$ (n,r) = (10,5)

19. The number of subsets of {1,2,3,…9} containing at least one odd number is **[EAMCET-2009]**

1) 324 2) 396 3) 496 4) 512

Ans: 3

Sol : No of subsets = $2^9 - 2^4$

= 512 – 16

= 496

20. ‘P’ points are chosen each of the three coplanar lines. The maximum number of triangles formed with vertices at these points is **[EAMCET-2009]**

1) $p3+3p^2$ 2) $\frac{1}{2}\left(p^3+p\right)$ 3) $\frac{p^2}{2}\left(5p-3\right)$ 4) $p^2\left(4p-3\right)$

Ans: 4

Sol : Let the lines be L_1, L_2, L_3

Max no of triangles = ${}^3c_2 \times {}^pc_2 \times {}^pc_1 + \left({}^pc_1\right)^3$

$= 6 \times \frac{p(p-1)}{2} \times p + p^3$

$= p^2\left(3p-3+p\right)$

$= p^2(4p-3)$

21. A binary sequence is an array of 0’s and 1’s the number of n-digit binary sequences which contain even number of 0’s is **[EAMCET-2009]**

1) 2^{n-1} 2) $2^{n}-1$ 3) $2^{n-1}-1$ 3) 2^{n}

Ans : 1

Sol : If n is even, no of n-digit binary sequences = 2^{n-1}.